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Small neutrino masses from structural cancellation in left-right symmetric model

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Abstract: The Type I, II and hybrid (I+II) seesaw mechanism, which explain why neutrinos are especially light, are consequences of the left-right symmetric model (LRSM). They can be classified by the ranges of parameters of LRSM. We show that a nearly cancellation between general Type-(I+II) seesaw is more natural than other types of seesaw in the LRSM if we consider their stability against radiative correction. In this scenario the small neutrino masses are due to the structure cancellation, and the masses of the right handed neutrino can be of order of O(10)TeV. The realistic model for non-zero neutrino masses, charged lepton masses and lepton tribimaximal mixing can be implemented by embedding A_4 flavor symmetry in the model with perturbations to the textures.

Keywords: Beyond Standard Model, Neutrino Physics, GUT.

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Contents

1. Introduction

The fact that neutrinos have very small masses has been established by a number of neutrino oscillation experiments [1] in the past decade, which is an important evident to go beyond the Standard Model. In order to generate very tiny neutrino masses, the very popular explanation is the seesaw mechanism [2].

In the so-called Type-I seesaw [2], extra very heavy Majorana right handed neutrinos (RHN) are introduced. When integrating them out, the neutrino mass is approximately $m_\nu \sim m_D^2/m_R$, so we assume that the m_D (the neutrino Dirac mass) is of the electroweak scale, i.e. $m_D \sim O(10^2\,GeV)$, we need the RHN mass $m_R \sim O(10^{16}\,GeV)$ which is hopeless to reach to direct test this mechanism. In the Type-II seesaw (triplet seesaw) [3], a heavy Higgs triplet Δ is introduced to play the similar role of heavy right handed neutrino to suppress the neutrino masses, we have $m_\nu \sim v^2/m_\Delta$, where $m_\Delta \sim O(10^{16}\,GeV)$ is the mass of Higgs triplet. In a general hybrid Type-(I+II) seesaw model, both terms make contributions to the neutrino masses. The crucial feature of such mechanisms are introducing heavy particles to suppress the neutrino masses, but the smallness of neutrino mass needs them to be too heavy to have any signals in future colliders.

Possible compromise between the impossible collider signals of such heavy particles and the smallness of neutrino masses is discussed in recent literatures in the framework of hybrid Type-(I+II) seesaw [4], where the small neutrino masses is from the structural cancellation, while suppression plays no role. In such scenario, the introduced heavy particles can be light enough to be direct produced in future colliders without violating the current bounds [5], so the possibilities have not been ruled out by experimental limits so far.

These types of seesaw are consequences of the left-right symmetric model (LRSM) [6], which is a possible extension of SM. In the model, unlike the SM that has only SU(2) left handed chiral matter, the right handed sector under Non-Abelian SU(2) representation are also introduced and correlated to the left handed sector. The LRSM not only leads to the seesaw mechanism but also provides explanation of the observed maximal P and C violation at low energy weak interaction, and is therefore likely in certain sense to be the final theory. The type of seesaw deduced from LRSM is determined by the space of parameters of the model. If we consider the stability of the parameters under the radiative correction, a model is "natural" if it is stable against the quantum correction, so fine-tuning for parameters is not needed. Before the mechanism can be tested directly in experiments, the naturalness is inevitable an important criteria for our model buildings.

In this paper, we will deduce the three types of seesaw in the LRSM and classify them by the ranges that the parameters locate. The 1-loop quantum correction of the parameter is evaluated and we find that the small neutrino mass from nearly cancellation in Type-(I+II) is more "natural" than other types of seesaw in the limit of small couplings in Higgs potential. So unlike the literature [4] where the cancellation relation is imposed by hands, the structure cancellation in LRSM is a natural result of the model. Therefore, in this scenario, the RHN can be light and be of order of O(10)TeV. Finally, non-zero neutrino masses, charged lepton masses and tribimaximal mixing [7] are generated by perturbations and embedding an extra A_4 [8] flavor symmetry into the model.

2. The model

2.1 The left-right symmetric model

The left-right symmetric model is based on the extended gauge group $G_{LR} = \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$, in which a Higgs bi-doublet Φ and left (right) Higgs triplet $\Delta_{L(R)}$ are introduced and with the representation assignments

$$\Phi \sim (2,2,0),\ \Delta_L \sim (3,1,2),\ \Delta_R \sim (1,3,2). \tag{2.1}$$

Under a discrete left-right symmetry, $l_L \leftrightarrow l_R^c$, $\Delta_L \leftrightarrow \Delta_R$ and $\Phi \leftrightarrow \Phi^T$, the invariant Lagrangian of the Yukawa interaction term is

$$-\mathcal{L} = y\bar{l}_L\Phi l_R + \tilde{y}\bar{l}_L\tilde{\Phi}l_R + \frac{1}{2}f[\bar{l}_L i\tau_2\Delta_L l_L^c + \overline{l_R^c}i\tau_2\Delta_R l_R] + \text{h.c.}, \tag{2.2}$$

where $l_{L(R)} = (\,\nu_{L(R)} \quad e_{L(R)}\,)^T$ is the lepton doublet, $\tilde{\Phi} = \tau_2\Phi^*\tau_2$, $l_{L(R)}^c \equiv C\overline{l_{L(R)}}^T$ with C being the charge-conjugation matrix. At first stage, the symmetry spontaneously broken into $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ by a non-zero vacuum expectation value (VEV) of Δ_R , leading to a heavy Majorana mass for right handed neutrinos. The second stage, the Φ develops VEV, breaking the symmetry to relic $\mathrm{U}(1)_{\mathrm{em}}$. The developed non-zero VEV consistent with $\mathrm{U}(1)_{\mathrm{em}}$ electromagnetic invariance are

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle\Phi\rangle = \begin{pmatrix} v & 0 \\ 0 & v' \end{pmatrix}. \tag{2.3}$$

The measurement of the ρ parameter [9] constrains the tree-level contribution of the Higgs triplet, $v_L \lesssim 1\,GeV$, which is much smaller than the electroweak scale $v \simeq 174\,GeV$, and we will work in the approximation $v' \ll v$. Integrating out the heavy fields the effective mass of neutrino can be written as the general Type-(I+II) seesaw formula

$$M_\nu \simeq M_L - M_D M_R^{-1} M_D^T = v_L f - \frac{v^2}{v_R} y f^{-1} y^T. \tag{2.4}$$

The dominant contribution from the first or second term determines the type of seesaw. In the model the charged lepton and Dirac neutrino mass matrix are simply obtained as $M_e = \tilde{y} v I$ and $M_D = y v I$ (I is the identity matrix), which we will discuss and implement by introducing flavor symmetry in section 4.

2.2 Higgs potential

Our aim here is to show the relations between the VEVs of the Higgs fields in LRSM, for this purpose, let us write the Higgs potential involving Φ and $\Delta_{L(R)}$. The most general renormalizable Higgs fields potential has the quadratic and quartic coupling terms and can not have any trilinear terms. So consistent with the transformation properties as eq. (2.1) and discrete left-right symmetry, the Higgs potential can be written as [10]

$$\begin{aligned} V(\Phi, \Delta_L, \Delta_R) =& -\mu_{ij}^2 tr[\Phi_i^\dagger \Phi_j] + \lambda_{ijkl} tr[\Phi_i^\dagger \Phi_j] tr[\Phi_k^\dagger \Phi_l] + \lambda'_{ijkl} tr[\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l] \\ & -\mu^2 tr[\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R] + \rho_1[(tr[\Delta_L^\dagger \Delta_L])^2 + (tr[\Delta_R^\dagger \Delta_R])^2] \\ & +\rho_2 \Big(tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L] + tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R] \Big) + \rho_3 tr[\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R] \\ & +\alpha_{ij} tr[\Phi_i^\dagger \Phi_j] \Big(tr[\Delta_L^\dagger \Delta_L] + tr[\Delta_R^\dagger \Delta_R] \Big) \\ & +\beta_{ij} \Big(tr[\Delta_L^\dagger \Delta_L \Phi_i \Phi_j^\dagger] + tr[\Delta_R^\dagger \Delta_R \Phi_i \Phi_j^\dagger] \Big) \\ & +\gamma_{ij} \Big(tr[\Delta_L^\dagger \Phi_i \Delta_R \Phi_j^\dagger] + h.c. \Big), \end{aligned} \tag{2.5}$$

where the sums over i, j, k and l run from 1 to 2, with $\Phi_1 = \Phi$ and $\Phi_2 = \tilde{\Phi}$. To recover the left-right symmetry and hermicity condition, the couplings satisfy the constraints,

$$\begin{array}{lll} \mu_{ij} = \mu_{ji}, & \lambda_{1212} = \lambda_{2121}, & \lambda_{iijk} = \lambda_{iikj}, \\ \lambda_{ijkk} = \lambda_{jikk}, & \lambda'_{ijkl} = \lambda'_{lijk} = \lambda'_{klij} = \lambda'_{jkli}, & \\ \alpha_{ij} = \alpha_{ji}, & \beta_{ij} = \beta_{ji}, & \gamma_{ij} = \gamma_{ji}. \end{array} \tag{2.6}$$

After the Higgs fields develop their VEV, we obtain

$$V = -\mu^2 (v_L^2 + v_R^2) + \frac{\rho}{4}(v_L^4 + v_R^4) + \frac{\rho'}{2} v_L^2 v_R^2 + \frac{\alpha}{2}(v_L^2 + v_R^2) v^2 + \gamma v_L v_R v^2, \tag{2.7}$$

where the approximation $v' \ll v$ is used, and the coefficients are

$$\begin{aligned} \gamma &= 2\gamma_{12}, \\ \alpha &= 2(\alpha_{11} + \alpha_{22} + \beta_{11}), \\ \rho &= 4(\rho_1 + \rho_2), \\ \rho' &= 2\rho_3. \end{aligned} \tag{2.8}$$

From the minimizing condition $\frac{\partial V}{\partial v_L} = \frac{\partial V}{\partial v_R} = 0$, if $v_L \neq v_R$, we get the relations for VEV of Higgs fields,

$$v_L v_R = \frac{\gamma}{\kappa} v^2, \tag{2.9}$$

where $\kappa = \rho - \rho'$. The mass m_L, m_R and m_D will be of order of v_L, v_R and v, respectively. In the next section, we will classify the types of seesaw mechanism generated from LRSM by the values of the ratio of Higgs particle self-couplings $\frac{\gamma}{\kappa}$.

3. The seesaw type and stability

We now discuss their contributions to the neutrino masses. Substituting the relation eq. (2.9) into the general Type-(I+II) seesaw formula eq. (2.4), we get

$$m_\nu = \left(f \left(\frac{\gamma}{\kappa} \right) - \frac{y^2}{f} \right) \frac{v^2}{v_R}. \tag{3.1}$$

According to the formula, following classification can be given.

1) Type-I seesaw: $f(\frac{\gamma}{\kappa}) \ll \frac{y^2}{f}$. It responds to the case of $m_\nu \simeq -\frac{y^2}{f}\frac{v^2}{v_R} = -m_D m_R^{-1} m_D^T$ dominant, the small neutrino mass is from the suppression of heavy v_R.

2) Type-II seesaw: $f(\frac{\gamma}{\kappa}) \gg \frac{y^2}{f}$. The term $m_\nu \simeq v_L f = m_L$ dominant, while $m_D m_R^{-1} m_D^T$ can be relatively neglected, i.e. the small neutrino mass is due to the smallness of v_L.

3) Nearly cancellation Type-(I+II) seesaw: $f(\frac{\gamma}{\kappa}) \simeq \frac{y^2}{f}$. The term m_L and $m_D m_R^{-1} m_D^T$ are comparable in magnitude and will nearly cancel their contributions to get small neutrino mass, we will see that this scenario is radiative stable.

However it is classical value at tree level, here we want to explore the behavior of the $\frac{\gamma}{\kappa}$ defined at the scale μ_0 under the radiative correction. The correction of γ and κ come from the 1-loop correction of the quartic coupling of operators $\Delta_L \Phi \Delta_R \Phi$ and $\Delta\Delta\Delta\Delta$, respectively. The renormalization group equation for γ and κ take the forms

$$\begin{aligned}
\mu \frac{d\gamma}{d\mu} &= \frac{1}{16\pi^2} \Big[(a_1 \alpha^2 + a_2 \beta^2 + a_3 \gamma^2) + (b_1 \alpha + b_2 \beta + b_3 \gamma) y^2 \\
&\qquad + (c_1 \alpha + c_2 \beta + c_3 \gamma) f^2 + (d_1 \alpha + d_2 \beta + d_3 \gamma) g^2 + e_1 g^4 + e_2 f^2 y^2 \Big], \\
\mu \frac{d\kappa}{d\mu} &= \frac{1}{16\pi^2} \Big[(a'_1 \rho_1^2 + a'_2 \rho_2^2 + a'_3 \rho_3^2) + (b'_1 \rho_1 + b'_2 \rho_2 + b'_3 \rho_3) f^2 \\
&\qquad + (c'_1 \rho_1 + c'_2 \rho_2 + c'_3 \rho_3) g^2 + d'_1 g^4 + d'_2 f^4 \Big],
\end{aligned} \tag{3.2}$$

in which the coefficients a, b, c, d, e are constants of order $O(1)$ that are determined by computing the corresponding 1-loop Feynman diagrams. $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \rho_i$ are coupling constants in Higgs potential eq. (2.5) and g the gauge coupling.

The Yukawa couplings f and y are of order $O(1)$, but the typical coupling constants in Higgs potential and the gauge coupling are generally assumed to be much smaller than that.

In fact, for large couplings, higher order or non-perturbative correction should be considered and we will not discuss them here. So we assume in this paper that in eq. (3.2) they can be approximately dropped, while only the loops that attribute to Yukawa couplings f, y play dominant role. We estimate the magnitude of the 1-loop corrections at scale μ to be

$$\delta\gamma \simeq \frac{-n_f f^2 y^2}{16\pi^2} \ln\left(\frac{\mu}{\mu_0}\right),$$
$$\delta\kappa \simeq \frac{-n_f f^4}{16\pi^2} \ln\left(\frac{\mu}{\mu_0}\right), \tag{3.3}$$

where n_f is the number of fermion species. The parameter $\frac{\gamma}{\kappa}$ is stable only when

$$0 = \delta\left(\frac{\gamma}{\kappa}\right) = \frac{(\delta\gamma)\kappa - \gamma(\delta\kappa)}{\kappa^2}, \tag{3.4}$$

so we get the relation

$$\frac{\gamma}{\kappa} \simeq \frac{y^2}{f^2}, \tag{3.5}$$

which is consistent with the nearly cancellation type $f(\frac{\gamma}{\kappa}) \simeq \frac{y^2}{f}$. In other words, if $m_\nu \simeq 0$ in eq. (3.1) arises from the cancellation between $f(\frac{\gamma}{\kappa})$ and $\frac{y^2}{f}$, because of eq. (3.4) it will lead to the stable value of $\frac{\gamma}{\kappa}$ that suppresses its radiative correction. Therefore, it is indicated that the scenario of nearly cancellation Type-(I+II) seesaw is more natural than other types of seesaw when we consider the factor of their stability. The neutrino mass is vanished when the cancellation relation $\frac{\gamma}{\kappa} = \frac{y^2}{f^2}$ is exactly hold as is shown in eq. (3.1).

The vanishing m_ν can also eliminate another unnaturalness that the texture of f is not uniquely determined in LRSM [11], e.g. if f is allowed, then so is $\hat{f} = \frac{m_\nu}{v_L} - f$. We can see that when $m_\nu = 0$, f is uniquely determined up to an unimportant phase or sign.

In this case, the v_R does not need to play the role of suppressing the neutrino mass, the RHN mass can be scale of $O(10)$TeV by the constraints of $v_L \lesssim 1\,\text{GeV}$. This possibility that v_R can be reachable TeV scale has not been ruled out by current bounds of experiments [5].

4. Non-zero neutrino masses and tribimaximal mixing

The textures of Yukawa matrices discussed above are simple, in which the Dirac neutrino masses and the ones coming from the left (right) Higgs triplet are degenerate,

$$M_D = yvI,$$
$$M_{L(R)} = fv_{L(R)}I. \tag{4.1}$$

The neutrino is massless when the cancellation relation is hold. However, the masses of neutrino are not trivially vanished. So we will discuss a deviation from this scenario by perturbations and introducing flavors symmetry to get a more realistic model.

We embed the extra A_4 symmetry [8] into LRSM by the assignments

$$l_{L(R)},\, l^c_{L(R)} \sim \underline{\mathbf{3}},\ \Phi \sim \underline{\mathbf{1}},\ \Delta_{L(R)} \sim \underline{\mathbf{1}}, \tag{4.2}$$

where, in A_4 group, $\underline{\mathbf{3}}$ stands for the real three-dimensional irreducible representation and $\underline{\mathbf{1}}$ for the trivial one in the three inequivalent one-dimensional representations $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$', $\underline{\mathbf{1}}$". So the invariant Yukawa Lagrangian for their couplings is

$$y(\overline{l_L}l_R)_{\underline{\mathbf{1}}}\Phi + \tilde{y}(\overline{l_L}l_R)_{\underline{\mathbf{1}}}\tilde{\Phi} + \frac{1}{2}i\tau_2 f\left((\overline{l_L}l_L^c)_{\underline{\mathbf{1}}}\Delta_L + (\overline{l_R^c}l_R)_{\underline{\mathbf{1}}}\Delta_R\right) + h.c., \tag{4.3}$$

in which the tensor product notations and properties of A_4 can be found in appendix A. Then the above assumptions eq. (4.1) as well as the lepton mass matrix $M_e = \tilde{y}vI$ can be achieved automatically, and they preserve the form of Higgs potential eq. (2.5) since the Higgs fields now are singlets of A_4.

In order to obtain non-trivial mixing, we need to introduce another scalar $\Sigma \sim \underline{\mathbf{3}}$ of A_4 to generate off-diagonal elements and assign the gauge group representation $\Sigma \sim (2,2,0)$ to it. The extra Higgs potential involving Σ and the couplings between Σ and $\Phi, \Delta_{L(R)}$ are list in the appendix B. The extra terms that contribute to the eq. (2.7) have no effect on the relation eq. (2.9), so the results in the previous sections are still valid.

Now the invariant Lagrangian of couplings between leptons and Σ is written as

$$h(\bar{l}_L l_R)_{\underline{\mathbf{3}}_s} \cdot \Sigma, \tag{4.4}$$

in which the subscript $\underline{\mathbf{3}}_s$ denotes the three dimensional symmetric tensor product as shown in appendix A. Expanding it into matrix in flavor basis we obtain the extra contributions

$$\begin{pmatrix} 0 & hv_{\Sigma_3} & hv_{\Sigma_2} \\ hv_{\Sigma_3} & 0 & hv_{\Sigma_1} \\ hv_{\Sigma_2} & hv_{\Sigma_1} & 0 \end{pmatrix}, \tag{4.5}$$

where $v_{\Sigma_i} = \langle \Sigma_i \rangle$. In the assumption of $v_{\Sigma_1} = v_{\Sigma_3} = 0$ and $hv_{\Sigma_2} = \delta \neq 0$, the matrix M_e and M_D have similar forms

$$M_e(M_D) = \tilde{y}(y)\ vI + \begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & 0 \\ \delta & 0 & 0 \end{pmatrix}. \tag{4.6}$$

Now, a deviation of M_e from M_D is needed by perturbations, in general the vanished elements will have non-zero values ϵ, and δ is perturbed to δ' and δ'',

$$M_e = \begin{pmatrix} \tilde{y}v & \epsilon_{12} & \delta'' \\ \epsilon_{21} & \tilde{y}v & \epsilon_{23} \\ \delta' & \epsilon_{32} & \tilde{y}v \end{pmatrix}. \tag{4.7}$$

We assume that $\epsilon_{21}, \epsilon_{32} \simeq \delta''$ and $\epsilon_{12}, \epsilon_{23} \simeq \delta'$, then we get the mass matrix of charged leptons that can be diagonalized by the unitary matrix

$$V_e = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{4.8}$$

in which $\omega = e^{\frac{2\pi i}{3}}$, i.e. $V_e^\dagger M_e V_e = diag(m_e, m_\nu, m_\tau)$, where

$$\begin{aligned} m_e &= \tilde{y}v + \delta' + \delta'', \\ m_\mu &= \tilde{y}v + (\omega\delta' + \omega^2\delta''), \\ m_\tau &= \tilde{y}v + (\omega^2\delta' + \omega\delta''). \end{aligned} \tag{4.9}$$

Under the condition of cancellation relation $\frac{\gamma}{\kappa} = \frac{y^2}{f^2}$ and non-diagonalized M_D eq. (4.6), a non-zero neutrino mass matrix now becomes

$$M_\nu = \Delta m I - \frac{2 f v_L \delta}{yv} \begin{pmatrix} \frac{\delta}{2yv} & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & \frac{\delta}{2yv} \end{pmatrix}, \tag{4.10}$$

where $\Delta m I$ is a perturbation. M_ν can be diagonalized by the unitary matrix

$$V_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}. \tag{4.11}$$

we get

$$M_\nu^{\text{diag}} = V_\nu^T M_\nu V_\nu = diag\left(\Delta m - \frac{2 f v_L \delta}{yv}\left(1 + \frac{\delta}{2yv}\right), \Delta m, \Delta m + \frac{2 f v_L \delta}{yv}\left(1 - \frac{\delta}{2yv}\right)\right). \tag{4.12}$$

The MNS matrix [12] is then obtained as

$$U_{\text{MNS}} = V_e^\dagger V_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{-i\pi/6}}{\sqrt{2}} \end{pmatrix}, \tag{4.13}$$

which is the tribimaximal mixing matrix up to a phase and hence fits the neutrino oscillation data well.

5. Conclusions

The Type I, II and hybrid (I+II) seesaw mechanisms can be deduced from the LRSM, and classified by the ranges of the parameter $\frac{\gamma}{\kappa}$, which represents the ratio of Higgs particle self-couplings. Assuming that the Yukawa coupling y, f are of order $O(1)$, then $\frac{\gamma}{\kappa} \ll \left(\frac{y}{f}\right)^2$ responds to Type-I seesaw, $\frac{\gamma}{\kappa} \gg \left(\frac{y}{f}\right)^2$ to Type-II seesaw and $\frac{\gamma}{\kappa} \simeq \left(\frac{y}{f}\right)^2$ to the comparable or nearly cancellation Type-(I+II) seesaw. In the limit of weak couplings in Higgs potential, we find that the parameter region $\frac{\gamma}{\kappa} \simeq \left(\frac{y}{f}\right)^2 \simeq O(1)$ is more stable against the radiative correction with respect to other regions, hence the nearly cancellation Type-(I+II) is more natural than other types of seesaw in the LRSM.

In the framework of nearly cancellation Type-(I+II) seesaw, the small neutrino masses arise from the cancellation between the contribution of the Type-I and Type-II. In this scenario, the RHN masses can be of order of O(10)TeV and be reachable in future colliders.

We give a realization of this kind of cancellation scenario by introducing an extra A_4 flavor symmetry to govern the textures of Yukawa coupling matrices. A realistic model that gives non-zero neutrino masses, charged lepton masses and lepton tribimaximal mixing is also implemented via introducing perturbations to the textures.

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A. Basic properties of A_4

The A_4 group has a real three dimensional irreducible representation $\underline{\mathbf{3}}$, and three inequivalent one dimensional representation $\underline{\mathbf{1}}, \underline{\mathbf{1}}', \underline{\mathbf{1}}''$, in which $\underline{\mathbf{1}}$ stands for the trivial representaion, and $\underline{\mathbf{1}}'$ and $\underline{\mathbf{1}}''$ are the non-trivial ones and complex conjugates to each other.

The multiplication rules of their non-trivial tensor products are given as

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} = \underline{\mathbf{3}}_s \oplus \underline{\mathbf{3}}_a \oplus \underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'' \quad \text{and} \quad \underline{\mathbf{1}}' \otimes \underline{\mathbf{1}}' = \underline{\mathbf{1}}'', \tag{A.1}$$

in which the subscript $s(a)$ stands for the symmetric (asymmetric) products. If we set $\psi_i, \phi_i \sim \underline{\mathbf{3}}$, then

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}} = \psi_1\phi_1 + \psi_2\phi_2 + \psi_3\phi_3, \tag{A.2}$$
$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}'} = \psi_1\phi_1 + \omega\psi_2\phi_2 + \omega^2\psi_3\phi_3, \tag{A.3}$$
$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}''} = \psi_1\phi_1 + \omega^2\psi_2\phi_2 + \omega\psi_3\phi_3, \tag{A.4}$$
$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_s} = (\psi_2\phi_3 + \psi_3\phi_2, \psi_3\phi_1 + \psi_1\phi_3, \psi_1\phi_2 + \psi_2\phi_1), \tag{A.5}$$
$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_a} = (\psi_2\phi_3 - \psi_3\phi_2, \psi_3\phi_1 - \psi_1\phi_3, \psi_1\phi_2 - \psi_2\phi_1), \tag{A.6}$$

with $\omega = e^{\frac{2\pi i}{3}}$.

B. Higgs potential

In addition to the A_4 singlet Higgs fields Φ and $\Delta_{L(R)}$, we have introduced another scalar $\Sigma \sim (2,2,0)(\underline{\mathbf{3}})$ under the group $G_{LR} \otimes A_4$, so the extra Higgs potential involving Σ should be added. The potential involving Φ and $\Delta_{L(R)}$ preserves its form eq. (2.5) since they are trivial representation of A_4, i.e. $\Phi, \Delta_{L(R)} \sim \underline{\mathbf{1}}$, we will not write them here again. According to the representation assignment of Σ, the invariant potential can be written as

$$\begin{aligned} V(\Sigma) = {} & \mu_\Sigma^2(\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}} + \lambda_1^\Sigma(\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}}(\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}} + \lambda_2^\Sigma(\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}'}(\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}''} \\ & + \lambda_3^\Sigma(\Sigma^\dagger\Sigma)_{\underline{\mathbf{3}}_s}(\Sigma^\dagger\Sigma)_{\underline{\mathbf{3}}_s} + \lambda_4^\Sigma(\Sigma^\dagger\Sigma)_{\underline{\mathbf{3}}_a}(\Sigma^\dagger\Sigma)_{\underline{\mathbf{3}}_a} \\ & + i\lambda_5^\Sigma(\Sigma^\dagger\Sigma)_{\underline{\mathbf{3}}_s}(\Sigma^\dagger\Sigma)_{\underline{\mathbf{3}}_a}, \end{aligned} \tag{B.1}$$

$$\begin{aligned} V(\Phi,\Sigma) = {} & \lambda_1^{\Phi\Sigma}(\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}} + \lambda_2^{\Phi\Sigma}(\Sigma^\dagger\Phi)_{\underline{\mathbf{3}}}(\Phi^\dagger\Sigma)_{\underline{\mathbf{3}}} \\ & + \lambda_3^{\Phi\Sigma}(\Sigma^\dagger\Phi)_{\underline{\mathbf{3}}}(\Sigma^\dagger\Phi)_{\underline{\mathbf{3}}} + h.c., \end{aligned} \tag{B.2}$$

$$\begin{aligned} V(\Delta_L,\Delta_R,\Sigma) = {} & \lambda_1^{\Delta\Sigma}[tr(\Delta_L^\dagger\Delta_L)_{\underline{\mathbf{1}}} + tr(\Delta_R^\dagger\Delta_R)_{\underline{\mathbf{1}}}](\Sigma^\dagger\Sigma)_{\underline{\mathbf{1}}} \\ & + \lambda_2^{\Delta\Sigma}\Sigma_{\underline{\mathbf{3}}}^\dagger\left([\Delta_L,\Delta_L^\dagger]_{\underline{\mathbf{1}}} + [\Delta_R,\Delta_R^\dagger]_{\underline{\mathbf{1}}}\right)\Sigma_{\underline{\mathbf{3}}}. \end{aligned} \tag{B.3}$$

There is no renormalizable term simultaneously involving $\Phi, \Delta_{L(R)}$ and Σ,

$$V(\Phi, \Delta_L, \Delta_R, \Sigma) = 0. \tag{B.4}$$

So the total Higgs potential is given by

$$V = V(\Phi, \Delta_L, \Delta_R) + V(\Sigma) + V(\Phi, \Sigma) + V(\Delta_L, \Delta_R, \Sigma). \tag{B.5}$$

References

[1] SNO collaboration, Q.R. Ahmad et al., *Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory*, *Phys. Rev. Lett.* **89** (2002) 011301 [nucl-ex/0204008];
C.K. Jung et al., *Oscillations of atmospheric neutrinos*, *Ann. Rev. Nucl. Part. Sci.* **51** (2001) 451;
KamLAND collaboration, K. Eguchi et al., *First results from KamLAND: evidence for reactor anti-neutrino disappearance*, *Phys. Rev. Lett.* **90** (2003) 021802 [hep-ex/0212021];
K2K collaboration, M.H. Ahn et al., *Indications of neutrino oscillation in a* 250*km long-baseline experiment*, *Phys. Rev. Lett.* **90** (2003) 041801 [hep-ex/0212007].

[2] P. Minkowski, $\mu \to e\gamma$ *at a rate of one out of* 1*-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
T. Yanagida, *Horizontal symmetry and masses of neutrinos* in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, O. Sawada and A. Sugamoto eds., KEK, Tsukuba Japan (1979);
M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, in *Supergravity*, F. van Nieuwenhuizen and D. Freedman eds., North Holland, Amsterdam The Netherlands (1979);
S.L. Glashow, *The future of elementary particle physics*, in *Quarks and leptons*, M. Lévy et al. eds., Plenum, New York U.S.A. (1980);
R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[3] J. Schechter and J.W.F. Valle, *Neutrino masses in* SU(2) × U(1) *theories*, *Phys. Rev.* **D 22** (1980) 2227;
T.P. Cheng and L.-F. Li, *Neutrino masses, mixings and oscillations in* SU(2) × U(1) *models of electroweak interactions*, *Phys. Rev.* **D 22** (1980) 2860;
M. Magg and C. Wetterich, *Neutrino mass problem and gauge hierarchy*, *Phys. Lett.* **B 94** (1980) 61;
R.N.Mohapatra and G.Senjanovic, *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165.

[4] W. Chao, S. Luo, Z.-z. Xing and S. Zhou, *A compromise between neutrino masses and collider signatures in the type-II seesaw model*, *Phys. Rev.* **D 77** (2008) 016001 [arXiv:0709.1069].

[5] F. del Aguila, J.A. Aguilar-Saavedra, A. Martinez de la Ossa and D. Meloni, *Flavour and polarisation in heavy neutrino production at* e^+e^- *colliders*, *Phys. Lett.* **B 613** (2005) 170 [hep-ph/0502189];
T. Han and B. Zhang, *Signatures for Majorana neutrinos at hadron colliders*, *Phys. Rev. Lett.* **97** (2006) 171804 [hep-ph/0604064];
F. del Aguila, J.A. Aguilar-Saavedra and R. Pittau, *Neutrino physics at large colliders*, *J. Phys. Conf. Ser.* **53** (2006) 506 [hep-ph/0606198];

S. Bray, J.S. Lee and A. Pilaftsis, *Resonant CP-violation due to heavy neutrinos at the LHC*, *Nucl. Phys.* **B 786** (2007) 95 [hep-ph/0702294];
F. del Aguila, J.A. Aguilar-Saavedra and R. Pittau, *Heavy neutrino signals at large hadron colliders*, *JHEP* **10** (2007) 047 [hep-ph/0703261].

[6] J.C. Pati and A. Salam, *Lepton number as the fourth color*, *Phys. Rev.* **D 10** (1974) 275 [*Erratum ibid.* **11** (1975) 703];
R.N. Mohapatra and J.C. Pati, *Left-right gauge symmetry and an isoconjugate model of CP-violation*, *Phys. Rev.* **D 11** (1975) 566; *A natural left-right symmetry*, *Phys. Rev.* **D 11** (1975) 2558;
G. Senjanović and R.N. Mohapatra, *Exact left-right symmetry and spontaneous violation of parity*, *Phys. Rev.* **D 12** (1975) 1502.

[7] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett.* **B 530** (2002) 167 [hep-ph/0202074];
Z.-Z. Xing, *Nearly tri-bimaximal neutrino mixing and CP-violation*, *Phys. Lett.* **B 533** (2002) 85 [hep-ph/0204049];
P.F. Harrison and W.G. Scott, *Symmetries and generalisations of tri-bimaximal neutrino mixing*, *Phys. Lett.* **B 535** (2002) 163 [hep-ph/0203209];
X.G. He and A. Zee, *Some simple mixing and mass matrices for neutrinos*, *Phys. Lett.* **B 560** (2003) 87 [hep-ph/0301092].

[8] E. Ma and G. Rajasekaran, *Softly broken A_4 symmetry for nearly degenerate neutrino masses*, *Phys. Rev.* **D 64** (2001) 113012 [hep-ph/0106291];
K.S. Babu, E. Ma and J.W.F. Valle, *Underlying A_4 symmetry for the neutrino mass matrix and the quark mixing matrix*, *Phys. Lett.* **B 552** (2003) 207 [hep-ph/0206292];
E. Ma, *A_4 symmetry and neutrinos with very different masses*, *Phys. Rev.* **D 70** (2004) 031901 [hep-ph/0404199]; *Aspects of the tetrahedral neutrino mass matrix*, *Phys. Rev.* **D 72** (2005) 037301 [hep-ph/0505209]; *Tetrahedral family symmetry and the neutrino mixing matrix*, *Mod. Phys. Lett.* **A 20** (2005) 2601 [hep-ph/0508099];
G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions*, *Nucl. Phys.* **B 720** (2005) 64 [hep-ph/0504165]; *Tri-bimaximal neutrino mixing, A_4 and the Modular Symmetry*, *Nucl. Phys.* **B 741** (2006) 215 [hep-ph/0512103];
K.S. Babu and X.-G. He, *Model of geometric neutrino mixing*, hep-ph/0507217;
A. Zee, *Obtaining the neutrino mixing matrix with the tetrahedral group*, *Phys. Lett.* **B 630** (2005) 58 [hep-ph/0508278];
M. Hirsch, A.S. Joshipura, S. Kaneko and J.W.F. Valle, *Predictive flavour symmetries of the neutrino mass matrix*, *Phys. Rev. Lett.* **99** (2007) 151802 [hep-ph/0703046].

[9] Particle Data Group collaboration, W.M. Yao et al., *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.

[10] R.N. Mohapatra and G. Senjanović, *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165;
J.F. Gunion, J. Grifols, A. Mendez, B. Kayser and F.I. Olness, *Higgs bosons in left-right symmetric models*, *Phys. Rev.* **D 40** (1989) 1546.

[11] E.K. Akhmedov and M. Frigerio, *Seesaw duality*, *Phys. Rev. Lett.* **96** (2006) 061802 [hep-ph/0509299].

[12] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.

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